Sheet 5

1. Determine if each of the following statements is true or not. If it is not, modify it to become true.
   a) When dealing with intensity values as random variable. Its variance is proportional to the image contrast.
   b) An intensity mapping on a 8 bit image has the form $s = T(z) = 2^8 - z$, where z is the pixel intensity in the input image and s is the pixel intensity in the output image, washes the image.
   c) In image processing terminology, image registration is the process of aligning input images to a master image so that comparison and measurements could be done across all the images.
   d) In digital image processing, when dealing with image transform, it is always possible to formulate the transform using matrix multiplications
   e) In digital image processing spatial operations are usually more computationally intensive compared to transform operation

2. Prove that the Fourier transform kernels are separable and symmetric

$$r(x, y, u, v) = e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$
$$s(x, y, u, v) = \frac{1}{MN} e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

3. Show that the two dimension transform with separable, symmetric kernels can be computed by (1) computing 1-D transform along the individual rows (columns) of the input, followed by (2) computing 1-D transform along the columns(rows) of the result of step (1)

4. a) An intensity mapping on a 8 bit image has the form $s = T(z) = 2^8 - z$, where z is the pixel intensity in the input image and s is the pixel intensity in the output image, has the effect of:
      I. Washing the image
      II. Increasing the contrast of the image
      III. Decreasing the contrast of the image
      IV. Getting the negative of the image
   b) When dealing with intensity values as random variable, positive third moment means
      I. Pixel values have bias to values higher than the mean
      II. Pixel values have bias to values smaller than the mean
      III. Pixel values have bias to be uniformly distributed around the mean
   c) When dealing with intensity values as random variable, negative third moment means
      I. Pixel values have bias to values higher than the mean
      II. Pixel values have bias to values smaller than the mean
      III. Pixel values have bias to be uniformly distributed around the mean

d) When dealing with intensity values as random variable, zero third moment means

   I. Pixel values have bias to values higher than the mean
   II. Pixel values have bias to values smaller than the mean
   III. Pixel values have bias to be uniformly distributed around the mean

e) In digital image processing, neighborhood and intensity mapping are

   I. Spatial operations
   II. Transform operations
   III. Spatial and transform operations

5. Geometric transformation is the mapping of the coordinates of each pixel in an input image to another (displaced/rotated,..) pixel in the output image. This mapping can be done in two different ways: Forward-mapping and inverse-mapping. Explain the two ways in contrasting the difference between them. Which way you prefer? Why?

6. What is image registration? Why and when they are necessary?

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sheet # 5

Q 5-1 Determine if each of the following is True or not, modify the false :

a) When dealing with intensity values as random variable. Its variance is proportional to the image contrast (True).

b) An intensity mapping on a 8-bit image has the form $s = T(z) = 2^8 - z$, where z is the pixel intensity in the input image and s is the pixel intensity in the output image, washes the image (False)

⇒ The true statement :
An intensity mapping on a 8 bit image ------, Performs the negative of the image

c) In image processing terminology, image registration is the process of aligning input images to a master image so that comparison and measurements could be done across all the images (True).

d) In digital image processing, when dealing with images transform it is always possible to formulate the transform using matrix multiplications. (False)

⇒ The true statement :
It is only possible to formulate the transform in terms of matrix multiplication if the following two Conditions apply :

1 * Both the forward and reverse are separable and symmetric.
2 * The operated image is square M by M.

e) In digital image processing spatial operations are usually more computationally intensive compared to transform operation (False)

⇒ The true statement:
In digital image processing transform operations are usually more computationally intensive compared to spatial operation

Q52: Prove that the Fourier transform kernels are separable and symmetric:

$$r(x,y,u,v) = e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

forward transformation kernel

$$s(x,y,u,v) = \frac{1}{MN}\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Inverse transformation kernel

Solution

$$\because\ r(x,y,u,v) = e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$r(x,y,u,v) = \underbrace{e^{-j2\pi\left(\frac{ux}{M}\right)}}\ \underbrace{e^{-j2\pi\left(\frac{vy}{N}\right)}}$$

$$r(x,y,u,v) = r_1(x,u)\cdot r_2(y,v) \quad \#$$

Then, the kernel is separable and it's also symmetric

the same form

* The same steps $s(x,y,u,v) = \frac{1}{M}\, e^{j2\pi\left(\frac{ux}{M}\right)}\ \frac{1}{N}\, e^{j2\pi\left(\frac{vy}{N}\right)}$ #

Q5.3: show that the two dimension transform with separable, symmetric kernels can be computed by (1) Computing 1-D transform along the individual rows (columns) of the input, followed by (2) Computing 1-D transform along the columns (rows) of the result of step (1)

using the following forward transform:

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$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\; r(x,y,u,v)$$

$$\because \quad r(x,y,u,v) = e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$\therefore T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}\right)} e^{-j2\pi\left(\frac{vy}{N}\right)}$$

$$= \sum_{x=0}^{M-1} e^{-j2\pi\left(\frac{ux}{M}\right)} \underbrace{\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{vy}{N}\right)}}_{T(x,v)}$$

along the columns

$$= \sum_{x=0}^{M-1} e^{-j2\pi\left(\frac{ux}{M}\right)}\, T(x,v)$$

$$\therefore \quad T(u,v) = \sum_{x=0}^{M-1} T(x,v)\, e^{-j2\pi\left(\frac{ux}{M}\right)}$$

along the rows

#

Q 54 Choose the Correct answer:

(a) An intensity mapping on a 8-bit image has the form $s = T(z) = 2^8 - z$, where z is the pixel intensity in the input Image and s is the pixel intensity in the output image, has the effect of:

i- washing the Image.
ii- Increasing the contrast of the image.
iii- Decreasing the Contrast of the image.
✓ (iv) Getting the negative of the image.

(b) When dealing with intensity values as random variable, positive third moment means:

✓ (i.) Pixel values have bias to values higher than the mean.
ii- Pixel values have bias to values smaller than the mean.
iii- Pixel values have bias to be uniformly distributed around the mean.

(c) When dealing with intensity values as random variable, negative third moment means:

i- Pixel values have bias to values higher than the mean.
✓ (ii) Pixel values have bias to values smaller than the mean.
iii- Pixel values have bias to be uniformly distributed around the mean

(d) When dealing with intensity values as random variable, zero third moment means:

✓ (iii) Pixel values have bias to be uniformaly distributed around the mean.

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(e) In digital image processing, neighborhood and intensity mapping are:

i. spatial operations.

ii. Transform operations.

iii. Spatial and Transform operations.

Q 5-5

Q 5-6

Report.

Tanta university
Faculty of engineering
Computer and automatic control department

Image processing course
Fourth year students
Sheet 05, Date : 02/11/2011

Sheet 5 solution

1.

Item	answer	The true statement if False
A	T	
B	F	An intensity mapping on a 8 bit image has the form $s = T(z) = 2^8 - z$, where z is the pixel intensity in the input image and s is the pixel intensity in the output image, performs the negative of the image.
C	T	
D	F	It is only possible to formulate the transform in terms of matrix multiplication if the following two conditions apply • Both the forward and the reverse kernels are separable and symmetric • The operated image is square M by M
E	F	In digital image processing transform operations are usually more computationally intensive compared to spatial operation since in most transform operation each output pixel value is calculated from all pixels in the input image. Also in most cases the process requires to do inverse transformations.

2.

$$\begin{aligned} r(x,y,u,v) &= e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})} \\ &= e^{-j2\pi(\frac{ux}{M})}e^{-j2\pi(\frac{vy}{N})} \\ &= r_1(x,u)r_2(y,v) \end{aligned}$$

Hence the kernel is separable
It's also symmetric (the same form for the two parts)
The steps can be applied to the reverse kernel: $s(x,y,u,v) = \frac{1}{MN}e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$

3. Using the forward transform and applying the separable property

$$\begin{aligned} T(u,v) &= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)r(x,y,u,v) \\ &= \sum_{x=0}^{M-1} r_1(x,u)\sum_{y=0}^{N-1} f(x,y)r_2(y,v) \\ &= \sum_{y=0}^{N-1} T(x,v)r_2(x,u) \end{aligned}$$

$T(x,v)$

Where

$$T(x,v) = \sum_{y=0}^{N-1} f(x,y) r_2(y,v)$$

For a fixed value of x, this equation is recognized as the 1-D transform along one row of f (x,y). By letting x vary from0 to M −1 we compute the entire array T(x,v). Then, by substituting this array into the last line of the previous equation we have the 1-D transform along the columns of T(x,v). In other words, when a kernel is separable, we can compute the 1-D transform along the rows of the image. Then we compute the 1-D transform along the columns of this intermediate result to obtain the final 2-D transform, T (u,v). We obtain the same result by computing the 1-D transform along the columns of f (x,y) followed by the 1-D transform along the rows of the intermediate result.

This result plays an important role in Chapter 4 when we discuss the 2-D Fourier transform. From Eq. (2.6-33), the 2-D Fourier transform is given by

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

Since the transform kernel is separable, we can write

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

$$T(u,v) = \sum_{x=0}^{M-1} e^{-j2\pi(\frac{ux}{M})} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{vy}{N})}$$

$$T(u,v) = \sum_{x=0}^{M-1} T(x,v)\, e^{-j2\pi(\frac{ux}{M})} = \sum_{x=0}^{M-1} T(x,v)\, r_1(x,u)$$

Where

$$T(x,v) = \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{vy}{N})}$$

4.

Item	Correct choice
A	IV
B	I
C	II
D	III
E	I

5. The two ways are:
   - Forward mapping: for each pixel in the input image, find its location in the output image and assign its value
     - Multiple output values are assigned to the same output pixel
     - Some output locations may not assigned values. Hence, interpolation is necessary to fill these locations.
   - Inverse mapping: for each pixel in the output image, find the location in the input image

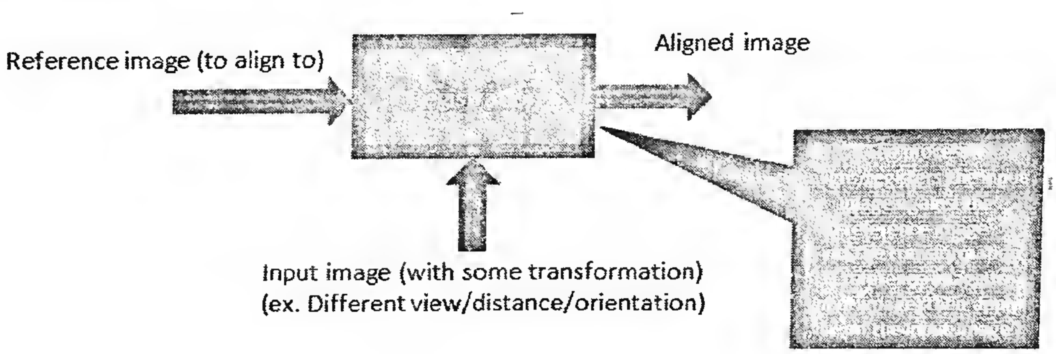
by applying the inverse transform, use the interpolation to calculate intensity value based on the intensities in the input image location.

I prefer doing inverse mapping is possible because the interpolation is directly calculated from the input image pixel values.

6. Image registration is the process of aligning different images taken for the same scene at different times (satellite image for monitoring earth movement) with different viewing angles/ distances. The objective is to align the images to some reference images so that measurements and comparison could be made across all the images.
A simple mode for registration is to build the mapping from a reference image f(x,y) to image g(v,w) using a set of tie points as follows:

$$x = c_1 v + c_2 w + c_3 vw + c_4$$
$$y = c_5 v + c_6 w + c_7 vw + c_8$$

By Applying these two equations at four **tie** points to obtain 8 equations in eight unknowns, solve them for the unknowns, we get the model. The model is then used to transform all input image pixels. After transforming the coordinates, interpolation is used to assign intensity values
The following figure explains image registration process.



Satellite images for monitoring the earth movement with distance & view

→ when subtracting two to get the difference we must align images first.